Indian Statistical Institute
B. Math. Hons. III Year
Semestral Examination 2002-2003
Optimization

Date:25-04-2003 Max. Marks: 120 Instructor: S. Ramasubramanian

Note: The paper carries 130 marks. Any score above 120 will be treated as 120.

1. Consider the linear programming problem: Minimize $(-3x_1 - 2x_2)$ subject to $x_1 + x_2 \leq 4, 2x_1 + x_2 \leq 6, x_1 \geq 0, x_2 \geq 0$.
(i) Use the simplex algorithm to solve the problem.
(ii) Can you solve the problem without using the simplex method? Justify your answer. [17+8=25]

2. Let P denote a linear programming problem in standard form.
(i) Show by an example that both P and its dual can be infeasible.
(ii) If P is unbounded below show that its dual is infeasible.
(iii) Is it possible for P to have exactly $k \geq 2$ extreme points and be unbounded below? [9+7+9=25]

3. Let Δ denote the standard $(n-1)$ dimensional simplex in R^n; let $a \in \Delta$ be an interior point and T_a be the corresponding projective transformation encountered in Karmarkar's algorithm.
(i) Show that T_a maps Δ onto Δ in a one-one fashion.
(ii) Show that $T_a(x)$ is a boundary point of $\Delta \Leftrightarrow x$ is a boundary point of Δ. [15+10=25]

4. Consider the problem: Find an $x \in R^n$ such that $Ax = b, x \geq 0$ if one exists; if not, report so.
Suppose entries of A, b are integers. Indicate how this problem can be converted to Karmarkar standard form. [20]

5. Consider the problem: Maximize $(x_1 + x_2 + \ldots + x_n)$ subject to the single constraint $(x_1^2 + x_2^2 + \ldots + x_n^2) = 1$.
(i) Justify the use of Lagrangean method for solving this problem.
(ii) Solve the problem using Lagrangean method. [15+10=25]

6. Show by an example that the Kuhn-Tucker conditions are not sufficient to characterize a local maximum in an inequality-constrained problem. [10]